

EJERCICIO A2BE 2222:

a) $\int x \cdot e^{2x} dx =$; b) $\int \frac{4+x}{3+x^2} dx =$

a) $\int \underbrace{x}_{u} \cdot \underbrace{e^{2x}}_{dv} dx =$
 $\int u dv = uv - \int v du$

$u = x \Rightarrow du = dx$

$\int dv = \frac{1}{2} \int 2 \cdot e^{2x} dx \Rightarrow v = \frac{1}{2} e^{2x}$

$= x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx =$

$= \frac{x \cdot e^{2x}}{2} - \frac{1}{2} \cdot \frac{1}{2} \int 2 e^{2x} =$

$= \frac{x \cdot e^{2x}}{2} - \frac{1}{4} e^{2x} = \boxed{\frac{e^{2x}}{2} \left(x - \frac{1}{2}\right) + C}$

b) $\int \frac{4+x}{3+x^2} dx = \underbrace{\int \frac{4}{3+x^2} dx}_{J_1} + \underbrace{\int \frac{x}{3+x^2} dx}_{J_2} = \dots$

$J_1 \Rightarrow \int \frac{4}{3+x^2} dx = 4 \int \frac{dx}{3+x^2} = \frac{4}{3} \int \frac{dx}{1 + \frac{x^2}{3}} =$

$= \frac{4}{3} \int \frac{dx}{1 + \left(\frac{x}{\sqrt{3}}\right)^2} = \frac{4\sqrt{3}}{3} \int \frac{\frac{1}{\sqrt{3}} dx}{1 + \left(\frac{x}{\sqrt{3}}\right)^2} =$

$$= \frac{4\sqrt{3}}{3} \operatorname{arctg} \frac{x}{\sqrt{3}}$$

$$\begin{aligned} \mathcal{J}_2 &\Rightarrow \int \frac{x}{3+x^2} dx = \frac{1}{2} \int \frac{2x}{3+x^2} dx = \\ &= \frac{1}{2} \ln(3+x^2) \end{aligned}$$

$$\int \frac{4+x}{3+x^2} dx = \dots = \boxed{\frac{4\sqrt{3}}{3} \operatorname{arctg} \frac{x}{\sqrt{3}} + \frac{1}{2} \ln|3+x^2| + C}$$