

$$\left(\sqrt[3]{\sqrt{125}}\right)^4 = \left(\sqrt[6]{5^3}\right)^4 = \sqrt[6]{5^{12}} = \sqrt[6]{5^6 \cdot 5^6} = 5^2 = \boxed{25}$$

$$\left(\sqrt{\sqrt{\frac{a^3 \cdot b^4}{c^2}}}\right)^3 = \sqrt[4]{\frac{a^9 \cdot b^{12}}{c^6}} = \frac{a^2 \cdot b^3}{c} \sqrt[4]{\frac{a}{c^2}} = \boxed{\frac{a^2 \cdot b^3}{c} \sqrt[4]{\frac{a}{c^2}}}$$

$$\sqrt{a^3 \sqrt[3]{a^2 \sqrt[4]{a^5}}} = \sqrt[6]{a^9 \cdot a^2 \sqrt[4]{a^5}} = \sqrt[6]{a^{11} \sqrt[4]{a^5}} = \sqrt[24]{a^{44} \cdot a^5} = \sqrt[24]{a^{49}} = \boxed{a^2 \sqrt[24]{a}}$$