

$$2^x + 2^y = 20$$

$$2^{x+y} = 64 \Rightarrow 2^{x+y} = 2^6 \Rightarrow x+y=6$$

$$y=6-x$$

$$2^x + 2^{6-x} = 20$$

$$2^x + \frac{2^6}{2^x} = 20$$

$$\text{c.v.} \\ 2^x = a$$

$$a + \frac{64}{a} = 20$$

$$\frac{a^2}{a} + \frac{64}{a} = \frac{20a}{a}$$

$$a^2 - 20a + 64 = 0$$

$$a = \frac{-(-20) \pm \sqrt{(-20)^2 - 4 \cdot 1 \cdot 64}}{2 \cdot 1} = \begin{matrix} \nearrow 4 \\ \searrow 16 \end{matrix}$$

$$2^x = a = 4 = 2^2 \Rightarrow \boxed{x=2 \rightarrow y=6-2=4}$$

$$2^x = a = 16 = 2^4 \Rightarrow \boxed{x=4 \rightarrow y=6-4=2}$$