

EJERCICIO M2BE2302 -J:

$$\int \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx =$$

$$\begin{aligned} x &= t^6 \\ dx &= 6t^5 dt \\ t &= \sqrt[6]{x} \end{aligned}$$

Conviene llamar $x=t^6$,
ya que 6 es el m.c.m.
de los índices de las raíces
de "x"

$$\int \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx = \int \frac{1}{\sqrt{t^6} + \sqrt[3]{t^6}} 6t^5 dt =$$

$$= \int \frac{6t^5 dt}{t^3 + t^2} = \int \frac{6t^5 dt}{t^2(t+1)} = 6 \int \frac{t^3}{t+1} dt = \dots$$

$$\begin{array}{r} \cancel{t^3} \\ -\cancel{t^3} - t^2 \\ \hline -t^2 \\ +t^2 + t \\ \hline t \\ -t - 1 \\ \hline -1 \end{array}$$

$$\dots = \int \left(t^2 - t + 1 + \frac{-1}{t+1} \right) dt =$$

$$= \frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1| =$$

$$= \frac{(\sqrt[6]{x})^3}{3} - \frac{(\sqrt[6]{x})^2}{2} + \sqrt[6]{x} - \ln|\sqrt[6]{x}+1| =$$

$$= \frac{\sqrt{x}}{3} - \frac{\sqrt[3]{x}}{2} + \sqrt[6]{x} - \ln|\sqrt[6]{x}+1| + C$$