

$$f(x) = ax^3 + bx^2 + cx + d$$

i) EXTREMO RELATIVO  $P(0,1)$   $\begin{cases} \textcircled{a} f(0) = 1 \\ \textcircled{b} f'(0) = 0 \end{cases}$

ii) PUNTO INFLEXIÓN  $Q(1,-1)$   $\begin{cases} \textcircled{c} f(1) = -1 \\ \textcircled{d} f''(1) = 0 \end{cases}$

$$\textcircled{a} f(0) = 1 \rightarrow \boxed{d = 1}$$

$$\textcircled{c} f(1) = -1 \rightarrow a + b + c + 1 = -1 \rightarrow a + b + c = -2$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$\textcircled{b} f'(0) = 0 \rightarrow \boxed{c = 0} \rightarrow \textcircled{a + b = -2}$$

$$f''(x) = 6ax + 2b$$

$$\textcircled{d} f''(1) = 0 \rightarrow \textcircled{6a + 2b = 0}$$

$$\begin{cases} a + b = -2 \xrightarrow{\cdot (-6)} -6a - 6b = 12 \\ 6a + 2b = 0 \end{cases}$$

$$\xrightarrow{\quad} \begin{cases} -6a - 6b = 12 \\ 6a + 2b = 0 \end{cases}$$

$$\hline -4b = 12 \rightarrow \boxed{b = -3}$$

$$a + (-3) = -2$$

$$\boxed{a = 1}$$

$$\boxed{f(x) = x^3 - 3x^2 + 1}$$