a)

$$\underbrace{\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ 0 & 2 \end{pmatrix}}_{A^t\ 2\times 3} \cdot \underbrace{\begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix}}_{A\ 2\times 3} = \underbrace{\begin{pmatrix} 5 & 2 & -2 \\ 2 & 1 & 0 \\ -2 & 0 & 4 \end{pmatrix}}_{A^t\cdot A\ 3\times 3}$$

$2\cdot 2-1\cdot(-1)$ $\quad 2\cdot 1-1\cdot 0$ $\quad 2\cdot 0-1\cdot 2$

$$|A^t \cdot A| = (20) - (4+16) = 0$$

b)

$$\begin{pmatrix} b & 0 \\ 1 & b \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2b & b & 0 \\ 2-b & 1 & 2b \end{pmatrix}$$

$$\begin{vmatrix} 2b & b \\ 2-b & 1 \end{vmatrix} = 2b - 2b + b^2 = b^2$$ Si $b\neq 0$ RANGO 2

$$\begin{vmatrix} 2b & 0 \\ 2-b & 2b \end{vmatrix} = 4b^2$$ Si $b\neq 0$ RANGO 2

$$\begin{vmatrix} b & 0 \\ 1 & 2b \end{vmatrix} = 2b^2$$ Si $b\neq 0$ RANGO 2

Si $b\neq 0$, RANGO DE BA ES 2.

Si $b=0$, RANGO $(BA) = 1$

$$BA = \begin{pmatrix} 0 & 0 & 0 \\ \boxed{2} & 1 & 0 \end{pmatrix}$$

c) $B = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

$$B^{-1} = \frac{(B^t)^{Adj.}}{|B|} = \frac{\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}^{Adj.}}{4} = \frac{\begin{pmatrix} 2 & 0 \\ -1 & 2 \end{pmatrix}}{4} = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

d) $B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$$B^2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$B^3 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

$$B^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} \Longrightarrow B^5 = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$$