

EJERCICIO A2BE3229:

"achimages.com"

Resuelve el sistema:

$$\left. \begin{aligned} 4X - 5Y &= \underbrace{\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}}_{M^{-1}} \underbrace{\begin{pmatrix} -9 & -4 & -6 \\ 17 & 3 & 13 \end{pmatrix}}_A \\ 6X + 4Y &= \underbrace{\begin{pmatrix} 10 & 4 & -10 \\ 12 & 10 & 22 \end{pmatrix}}_B \end{aligned} \right\}$$

Tenemos que hallar $\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^{-1}$
es decir, la inversa de M , siendo M :

$$M = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$M^{-1} = \frac{(M^t)^{Adj}}{|M|} = \frac{\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^{Adj}}{\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}}} = \frac{\begin{pmatrix} 2 & -(-1) \\ -(-1) & 1 \end{pmatrix}}{1} \Rightarrow$$

$$M^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

El producto de matrices de la 1ª ecuación:

$$\underbrace{\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}}_{2 \times 2} \cdot \underbrace{\begin{pmatrix} -9 & -4 & -6 \\ 17 & 3 & 13 \end{pmatrix}}_{2 \times 3} = \underbrace{\begin{pmatrix} -18+17 & -8+3 & -12+13 \\ -9+17 & -4+3 & -6+13 \end{pmatrix}}_{2 \times 3} =$$
$$= \begin{pmatrix} -1 & -5 & 1 \\ 8 & -1 & 7 \end{pmatrix}$$

"A"

El sistema queda:

$$\left. \begin{aligned} (1) \quad 4X - 5Y &= \begin{pmatrix} -1 & -5 & 1 \\ 8 & -1 & 7 \end{pmatrix} \\ (2) \quad 6X + 4Y &= \begin{pmatrix} 10 & 4 & -10 \\ 12 & 10 & 22 \end{pmatrix} \end{aligned} \right\} \begin{aligned} (x3) \quad 12X - 15Y &= \begin{pmatrix} -3 & -15 & 3 \\ 24 & -3 & 21 \end{pmatrix} \\ (-) \quad (x2) \quad 12X + 8Y &= \begin{pmatrix} 20 & 8 & -20 \\ 24 & 20 & 44 \end{pmatrix} \end{aligned}$$

$$-23Y = \begin{pmatrix} -23 & -23 & 23 \\ 0 & -23 & -23 \end{pmatrix}$$

$$Y = \frac{1}{-23} \begin{pmatrix} -23 & -23 & 23 \\ 0 & -23 & -23 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}} = Y$$

De la ecuación (2):

$$6X + 4Y = \begin{pmatrix} 10 & 4 & -10 \\ 12 & 10 & 22 \end{pmatrix}$$

$$6X + 4 \cdot \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 4 & -10 \\ 12 & 10 & 22 \end{pmatrix}$$

$$6X + \begin{pmatrix} 4 & 4 & -4 \\ 0 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 4 & -10 \\ 12 & 10 & 22 \end{pmatrix}$$

$$6X = \begin{pmatrix} 10 & 4 & -10 \\ 12 & 10 & 22 \end{pmatrix} - \begin{pmatrix} 4 & 4 & -4 \\ 0 & 4 & 4 \end{pmatrix}$$

$$6X = \begin{pmatrix} 6 & 0 & -6 \\ 12 & 6 & 18 \end{pmatrix}$$

$$X = \frac{1}{6} \begin{pmatrix} 6 & 0 & -6 \\ 12 & 6 & 18 \end{pmatrix}$$

$$\boxed{X = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{pmatrix}}$$