

EJERCICIO N2BE3348:

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$$\int \frac{x+1}{x^2+x+1} =$$

$$x^2+2x+1 = (x+1)^2$$

$$x^2+x+1 \Rightarrow \left(x+\frac{1}{2}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{2} + \frac{1}{4}$$

$$x^2+x+\frac{1}{4} = \left(x+\frac{1}{2}\right)^2$$

$$x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\frac{1}{2} \int \frac{2(x+1)}{x^2+x+1} dx = \frac{1}{2} \int \frac{2x+2}{x^2+x+1} dx =$$

$$= \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx + \frac{1}{2} \int \frac{1}{x^2+x+1} dx$$

$$J_1 = \frac{1}{2} \ln|x^2+x+1|$$

J_2

$$J_2 = \int \frac{1}{x^2+x+1} dx = \int \frac{1}{\frac{3}{4} + \left(x+\frac{1}{2}\right)^2} dx =$$

$$= \frac{1}{\frac{3}{4}} \int \frac{1}{1 + \frac{\left(x+\frac{1}{2}\right)^2}{\frac{3}{4}}} dx = \frac{4}{3} \int \frac{1}{1 + \left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2} dx =$$

$$= \frac{4}{3} \int \frac{1}{1 + \left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}}\right)^2} dx =$$

$$= \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \int \frac{\frac{2}{\sqrt{3}}}{1 + \left(\frac{2x+1}{\sqrt{3}}\right)^2} dx =$$

$$= \frac{2\sqrt{3}}{3} \operatorname{arctg} \frac{2x+1}{\sqrt{3}}$$

$$\int \frac{x+1}{x^2+x+1} dx = \frac{1}{2} \ln|x^2+x+1| + \frac{1}{2} \cdot \frac{2\sqrt{3}}{3} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} + C$$