

EJERCICIO H2B53350:

"achimager.com"

$$\int \frac{1}{x^2 - 3x + 3} dx =$$

Difícil de ver, pero hay que intentarlo
probando diferentes opciones:

$$x^2 - 3x + 3 = \left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$$

$$\boxed{\int \frac{f'(x)}{1+[f(x)]^2} dx = \arctan f(x) + C}$$

$$\begin{aligned} \left(x - \frac{3}{2}\right)^2 &= x^2 - 2 \cdot x \cdot \frac{3}{2} + \frac{9}{4} \\ &= x^2 - 3x + \frac{9}{4} \end{aligned}$$

$$\left(\frac{9}{4} + \frac{3}{4} = \frac{12}{4} = 3\right)$$

Por ello:

$$\begin{aligned} \int \frac{1}{x^2 - 3x + 3} dx &= \int \frac{dx}{\frac{3}{4} + \left(x - \frac{3}{2}\right)^2} = \\ &= \frac{1}{\frac{3}{4}} \int \frac{dx}{1 + \frac{\left(x - \frac{3}{2}\right)^2}{\frac{3}{4}}} = \frac{4}{3} \int \frac{dx}{1 + \frac{4\left(x - \frac{3}{2}\right)^2}{3}} = \\ &= \frac{4}{3} \int \frac{dx}{1 + \left[\frac{2\left(x - \frac{3}{2}\right)}{\sqrt{3}}\right]^2} = \frac{4}{3} \int \frac{dx}{1 + \left(\frac{2x-3}{\sqrt{3}}\right)^2} = \\ &= \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \int \frac{\frac{2}{\sqrt{3}} dx}{1 + \left(\frac{2x-3}{\sqrt{3}}\right)^2} = \boxed{\frac{2\sqrt{3}}{3} \arctan \frac{2x-3}{\sqrt{3}} + C} \end{aligned}$$