EJERCICIO A2BE3446:

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$$M_k = \begin{pmatrix} k & 0 & k \\ 0 & k & 0 \\ 2k-3 & 0 & k \end{pmatrix}$$

a) Rango M_k?

b) M_1 para $k=1$, $B = \begin{pmatrix} -4 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

Hallar $X \Rightarrow X \cdot M_1 + X M_1^t = B$

a) Rango M_k?

$$|M_k| = \begin{vmatrix} k & 0 & k \\ 0 & k & 0 \\ 2k-3 & 0 & k \end{vmatrix} = k^3 - k^2(2k-3) =$$

$$= k^3 - 2k^3 + 3k^2 = -k^3 + 3k^2$$

$$-k^3 + 3k^2 = 0$$

$$k^2(-k+3) = 0 \begin{cases} k^2 = 0 \Rightarrow \boxed{k=0} \\ -k+3 = 0 \Rightarrow \boxed{k=3} \end{cases}$$

Si $k \neq 0$ y $k \neq 3$ $R(M_k) = 3$

Si $k=0 \Rightarrow R(M_k) < 3$, hallamos

$$M_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -3 & 0 & 0 \end{pmatrix}$$

$R(M_0) = 1$; Si $k=0$

Si $k=3$

$$M_3 = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$$

$R(M_3) = 2$; si $k=3$

b) Si $k=1$

$$M_1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} -4 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$X \cdot M_1 + X \cdot M_1^t = B$$

$$X\left(M_1+M_1^t\right)_1 = B_1$$

$$X\left(M_1+M_1^t\right)\cdot\left(M_1+M_1^t\right)^{-1} = B\left(M_1+M_1^t\right)^{-1}$$

$$X\cdot I = B\cdot\left(M_1+M_1^t\right)^{-1}$$

$$\boxed{X = B\cdot\left(M_1+M_1^t\right)^{-1}}$$

$$M_1+M_1^t = \begin{pmatrix}1&0&1\\0&1&0\\-1&0&1\end{pmatrix}+\begin{pmatrix}1&0&-1\\0&1&0\\1&0&1\end{pmatrix}=\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}$$

$$\left(M_1+M_1^t\right)^{-1} = \frac{\left[\left(M_1+M_1^t\right)^t\right]^{Adj}}{\left|M_1+M_1^t\right|} = \frac{\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}^{Adj}}{8} =$$

$$= \frac{\begin{pmatrix}\begin{vmatrix}2&0\\0&2\end{vmatrix} & -\begin{vmatrix}0&0\\0&2\end{vmatrix} & \begin{vmatrix}0&2\\0&0\end{vmatrix}\\ -\begin{vmatrix}0&0\\0&2\end{vmatrix} & \begin{vmatrix}2&0\\0&2\end{vmatrix} & -\begin{vmatrix}2&0\\0&0\end{vmatrix}\\ \begin{vmatrix}0&0\\2&0\end{vmatrix} & -\begin{vmatrix}2&0\\0&0\end{vmatrix} & \begin{vmatrix}2&0\\0&2\end{vmatrix}\end{pmatrix}}{8} = \frac{\begin{pmatrix}4&0&0\\0&4&0\\0&0&4\end{pmatrix}}{8} =$$

$$= \frac{1}{8}\begin{pmatrix}4&0&0\\0&4&0\\0&0&4\end{pmatrix} = \boxed{\begin{pmatrix}1/2&0&0\\0&1/2&0\\0&0&1/2\end{pmatrix} = \left(M_1+M_1^t\right)^{-1}}$$

Entonces:

$$X = B\cdot\left(M_1+M_1^t\right)^{-1} = \underset{2\times3}{\begin{pmatrix}-4&2&0\\0&0&4\end{pmatrix}}\cdot\underset{3\times3}{\begin{pmatrix}1/2&0&0\\0&1/2&0\\0&0&1/2\end{pmatrix}} = \underset{2\times3}{\begin{pmatrix}-2&1&0\\0&0&2\end{pmatrix}}$$

$$\boxed{X = \begin{pmatrix}-2&1&0\\0&0&2\end{pmatrix}}$$