EJERCICIO F2BE2096:

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- $f = 0{,}25\ Hz$
- $d = 2\ m$
- Sentido positivo Ox
- En $x = 0{,}5$; $t = 2\ s \Rightarrow y = 0$; $v_{oscilación}$ negativa
  $t = 3\ s \Rightarrow y = -0{,}2\ m$

a) Ecuación de onda?
b) $v_{máx}$ de oscilación y $v_{propagación}$?
c) Diferencia de fase si $d = 0{,}75\ m$?

a) Ecuación de la onda:

sentido positivo Ox

$$y(x,t) = \frac{A}{?} \cdot \operatorname{sen}\left(\frac{\omega}{?} t - \frac{k}{?} x + \frac{\varphi_0}{?}\right)$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{1}{f} = \frac{1}{0{,}25} = 4\ s$$

$$\omega = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}\ rad/s = \omega}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \boxed{\pi\ rad/m = k}$$

$$y(x,t) = A \cdot \operatorname{sen}\left(\frac{\pi}{2} \cdot t - \pi x + \varphi_0\right)$$

φ_0: fase inicial:

En $x = 0{,}5\ m$; $t = 2\ s \Rightarrow y = 0$; $v < 0$

$$0 = A \cdot \operatorname{sen}\left(\frac{\pi}{2} \cdot 2 - \pi \cdot 0{,}5 + \varphi_0\right)$$

$$0 = A \cdot \operatorname{sen}\left(\pi - \frac{\pi}{2} + \varphi_0\right)$$

$$0 = \operatorname{sen}\left(\frac{\pi}{2} + \varphi_0\right)$$

$$\left(\frac{\pi}{2} + \varphi_0\right) \nearrow 0 \Rightarrow \frac{\pi}{2} + \varphi_0 = 0 \Rightarrow \boxed{\varphi_0 = -\frac{\pi}{2}}$$

$$\left(\frac{\pi}{2} + \varphi_0\right) \searrow \pi \Rightarrow \frac{\pi}{2} + \varphi_0 = \pi \Rightarrow \boxed{\varphi_0 = \frac{\pi}{2}}$$

$$\boxed{\text{¿ } \varphi_0 = -\frac{\pi}{2} \text{ ó } \varphi_0 = \frac{\pi}{2} \text{ ?}}$$

Como dice que la velocidad es negativa en $x = 0{,}5\ m$; $t = 2s$

$$v(x,t) = \frac{dy(x,t)}{dt} = A \cdot \frac{\pi}{2} \cos\left(\frac{\pi}{2}t - \pi x + \varphi_0\right)$$

Si $\varphi_0 = -\frac{\pi}{2}$

$$v(0{,}5;2) = A \cdot \frac{\pi}{2} \cdot \cos\left(\frac{\pi}{2}\cdot 2 - \pi \cdot 0{,}5 - \frac{\pi}{2}\right) =$$

$$= A \cdot \frac{\pi}{2} \cos\left(\pi - \frac{\pi}{2} - \frac{\pi}{2}\right) =$$

$$v = A \cdot \frac{\pi}{2} \underbrace{\cos 0}_{1} > 0 \quad (\text{No negativa})$$

Si $\varphi_0 = \frac{\pi}{2}$

$$v(0{,}5;2) = A \cdot \frac{\pi}{2} \cos\left(\frac{\pi}{2}\cdot 2 - \pi \cdot 0{,}5 + \frac{\pi}{2}\right) =$$

$$= A \cdot \frac{\pi}{2} \cos\left(\pi - \frac{\pi}{2} + \frac{\pi}{2}\right) =$$

$$= A \cdot \frac{\pi}{2} \cos \pi < 0 \quad (\text{Si negativa})$$

$\varphi_0 = \frac{\pi}{2}$ si $v < 0$

Respecto a la Amplitud:

En $x = 0{,}5\ m$; $t = 3s \Rightarrow y = -0{,}2\ m$

$$y(x,t) = A \cdot \operatorname{sen}\left(\frac{\pi}{2}\cdot t - \pi x + \frac{\pi}{2}\right)$$

$$-0{,}2 = A \cdot \operatorname{sen}\left(\frac{\pi}{2}\cdot 3 - \pi \cdot 0{,}5 + \frac{\pi}{2}\right)$$

$$-0{,}2 = A \cdot \operatorname{sen}\left(\frac{3\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$-0{,}2 = A \cdot \operatorname{sen}\left(\frac{3\pi}{2}\right)$$

$$-0{,}2 = A \cdot (-1) \Rightarrow \boxed{A = 0{,}2\ m}$$

Ecuación de onda completa:

$$\boxed{y(x,t) = 0{,}2 \cdot \operatorname{sen}\left(\frac{\pi}{2}t - \pi x + \frac{\pi}{2}\right)}$$

b) Velocidad máxima oscilación y Vpropagación:

$$v(x,t) = \frac{dy(x,t)}{dt} = 0{,}2 \cdot \frac{\pi}{2} \cdot \cos\left(\frac{\pi}{2}t - \pi x + \frac{\pi}{2}\right)$$

igual a 1 si máx

$$v_{máx} = 0{,}2 \cdot \frac{\pi}{2} = 0{,}1\,\pi \text{ m/s}$$

$$v_{propagación} = \frac{\lambda}{T} = \frac{2}{4} = \frac{1}{2} \Rightarrow \boxed{v_{prop} = 0{,}5 \text{ m/s}}$$

c) Diferencia de fase, mismo instante, d = 0,75 m:

$$\Delta\varphi = \varphi_2 - \varphi_1 \quad \text{mismo instante}$$

$$\Delta\varphi = \left(\frac{\pi}{2}t - \pi x + \frac{\pi}{2}\right)_2 - \left(\frac{\pi}{2}t - \pi x + \frac{\pi}{2}\right)_1$$

$$\Delta\varphi = -\pi x_2 + \pi x_1$$

$$\Delta\varphi = -\pi(\underbrace{x_2 - x_1}_{0{,}75\text{ m}})$$

$$\Delta\varphi = -0{,}75 \cdot \pi = -2{,}355 \text{ rad}$$

$$\boxed{|\Delta\varphi| = 2{,}355 \text{ rad}}$$

Con la fórmula:

$$\Delta\varphi = k \cdot \Delta x = \pi \cdot 0{,}75$$
$$\Delta\varphi = 0{,}75\,\pi \text{ rad}$$