EJERCICIO M1BE24J1:

EBAU CANARIAS JULIO 2018

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Determinar a y b para que

$f(x) = a\cdot\sqrt{3x+3} + b\cdot\sqrt{x-1}$

tenga un Punto de Inflexión en $(2,8)$

Por tener un P.I. en $(2,8)$:

(I) $f(2)=8$

(II) $f''(2)=0$

De (I): $8 = a\cdot\sqrt{3\cdot 2+3} + b\sqrt{2-1}$

$$\boxed{8 = 3a+b}$$

(II): Necesitamos $f''(x)$

$$f'(x) = a\cdot\frac{3}{2\sqrt{3x+3}} + b\frac{1}{2\sqrt{x-1}} =$$

$$= \frac{3a}{2\sqrt{3x+3}} + \frac{b}{2\sqrt{x-1}}$$

$$f''(x) = \frac{0\cdot 2\sqrt{3x+3} - \left(2\sqrt{3x+3}\right)'\cdot 3a}{\left(2\sqrt{3x+3}\right)^2} + \frac{0\cdot 2\sqrt{x-1} - \left(2\sqrt{x-1}\right)'\cdot b}{\left(2\sqrt{x-1}\right)^2} =$$

$$= \frac{-\cancel{2}\cdot\frac{3}{\cancel{2}\sqrt{3x+3}}\cdot 3a}{4(3x+3)} + \frac{-\cancel{2}\,\frac{1}{\cancel{2}\sqrt{x-1}}\cdot b}{4(x-1)} =$$

$$= \frac{-9a}{4(3x+3)\sqrt{3x+3}} + \frac{-b}{4(x-1)\sqrt{x-1}}$$

(II) $f''(2)=0 \Rightarrow f''(2) = \frac{-\cancel{9}a}{4\cdot\cancel{9}\cdot 3} - \frac{b}{4\cdot 1\cdot 1} = \frac{-a}{12} - \frac{b}{4}$

$$-\frac{a}{12} - \frac{b}{4} = 0 \Rightarrow \frac{-a-3b}{12} = 0 \Rightarrow -a-3b=0$$

$$\boxed{a+3b=0}$$

Con las dos ecuaciones:

$$\left.\begin{array}{r} 3a+b=8 \\ a+3b=0 \end{array}\right\} \begin{array}{l} \ \ \cancel{3a}+b=8 \\ (\times 3)\ -\cancel{3a}+9b=0 \\ \hline \end{array}$$

Restando: $-8b=8 \Rightarrow \boxed{b=-1}$

$a-3=0 \Rightarrow \boxed{a=3}$