

EJERCICIO M1BE3292:

"achiimagec.com"

Calcula el valor de los siguientes logaritmos (sin calculadora):

$$a) \boxed{\log_3 81} = \log_3 3^4 = 4 \cdot \overbrace{\log_3 3}^1 = \boxed{4}$$

$$b) \boxed{\log_3 (3 \cdot \sqrt{3})} = \log_3 3 + \log_3 \sqrt{3} = \\ = \overbrace{\log_3 3}^1 + \log_3 3^{1/2} = 1 + \frac{1}{2} \log_3 3 = \\ = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$$

$$c) \boxed{\log_3 \frac{\sqrt[4]{3}}{9}} = \log_3 \sqrt[4]{3} - \log_3 9 = \\ = \log_3 3^{1/4} - \log_3 3^2 = \frac{1}{4} \overbrace{\log_3 3}^1 - 2 \overbrace{\log_3 3}^1 = \\ = \frac{1}{4} - 2 = \frac{1-8}{4} = \boxed{\frac{-7}{4}}$$

$$d) \boxed{\log_{81} 3} = \log_{81} \sqrt[4]{81} = \text{¡ } 3 = \sqrt[4]{81} = \sqrt[4]{3^4}! \\ = \log_{81} 81^{1/4} = \frac{1}{4} \overbrace{\log_{81} 81}^1 = \boxed{\frac{1}{4}}$$

$$e) \boxed{\log_{81} \left(-\frac{1}{9}\right)} = \boxed{\text{No existe}}$$

$$f) \boxed{\log_{81} \frac{\sqrt{3}}{3}} = \log_{81} \frac{\sqrt[4]{81}}{\sqrt[4]{81}} = \log_{81} \frac{\sqrt[8]{81}}{\sqrt[4]{81}} = \\ = \log_{81} \frac{\sqrt[8]{81}}{\sqrt[8]{81^2}} = \log_{81} \sqrt[8]{\frac{81}{81^2}} = \\ = \log_{81} \sqrt[8]{\frac{1}{81}} = \log_{81} \sqrt[8]{81^{-1}} = \log_{81} 81^{-1/8} = \\ = -\frac{1}{8} \cdot \overbrace{\log_{81} 81}^1 = \boxed{-\frac{1}{8}}$$

$$\begin{aligned}
 g) \quad \boxed{\log_{1/9} \frac{\sqrt[4]{3}}{3}} &= \log_{1/9} \frac{\sqrt[4]{\sqrt{9}}}{\sqrt{9}} = \log_{1/9} \frac{\sqrt[8]{9}}{\sqrt{9}} = \\
 &= \log_{1/9} \frac{\sqrt[8]{9}}{\sqrt[8]{9^4}} = \log_{1/9} \sqrt[8]{\frac{9}{9^4}} = \log_{1/9} \sqrt[8]{\frac{1}{9^3}} = \\
 &= \log_{1/9} \sqrt[8]{9^{-3}} = \log_{1/9} 9^{-3/8} = \log_{1/9} \left(\frac{1}{9}\right)^{3/8} = \\
 &= \frac{3}{8} \cdot \overbrace{\log_{1/9} \left(\frac{1}{9}\right)}^1 = \boxed{\frac{3}{8}}
 \end{aligned}$$