

EJERCICIO M2B53470:

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Resuelve la ecuación matricial:

$$XA + B^t = A \cdot B$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 0 & 0 & -1 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$X \cdot A + B^t = A \cdot B$$

$$X \cdot A + B^t - B^t = A \cdot B - B^t$$

$\rightarrow$ matriz nula (elemento neutro de la suma)

$$X \cdot A + 0 = A \cdot B - B^t$$

$$X \cdot A = A \cdot B - B^t$$

$$X \cdot A \cdot A^{-1} = (A \cdot B - B^t) \cdot A^{-1}$$

$\rightarrow$ matriz identidad (elemento neutro del producto)

$$X \cdot I = (A \cdot B - B^t) \cdot A^{-1}$$

$$X = (A \cdot B - B^t) \cdot A^{-1}$$

$$A \cdot B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 2 \cdot 1 + 3 \cdot (-1) & 7 & 5 \\ 1 \cdot 1 - 1 \cdot 1 - 2 \cdot 1 & 1 & 1 \\ 0 \cdot 1 + 0 \cdot 1 - 1 \cdot 1 & 0 & 1 \end{pmatrix} =$$

$$A \cdot B = \begin{pmatrix} 0 & 7 & 5 \\ -2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$B^t = \begin{pmatrix} 1 & 1 & -1 \\ 3 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{(A^t)^{Adj}}{|A|} = \dots$$

Continúa...

$$\boxed{A^{-1}} = \frac{(A^t)^{\Delta \text{adj}}}{|A|} = \frac{\begin{pmatrix} 1 & 1 & 0 \\ 2 & -1 & 0 \\ 3 & 2 & 1 \end{pmatrix}^{\Delta \text{adj}}}{\begin{vmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{vmatrix}} =$$

$$= \frac{\begin{pmatrix} \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \end{pmatrix}}{-1-2} = \frac{\begin{pmatrix} -1 & -2 & 7 \\ -1 & 1 & 1 \\ 0 & 0 & -3 \end{pmatrix}}{-3} = \boxed{\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{7}{3} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix}}$$

$$X = (A \cdot B - B^t) \cdot A^{-1} =$$

$$\boxed{X} = \left[\begin{pmatrix} 0 & 7 & 5 \\ -2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 & -1 \\ 3 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \right] \cdot \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{7}{3} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & 6 & 6 \\ -5 & -1 & 1 \\ -1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{7}{3} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 5/3 & -8/3 & 19/3 \\ -2 & -3 & 13 \\ -\frac{2}{3} & -\frac{1}{3} & \frac{8}{3} \end{pmatrix}}$$